Anomalous Frequency Scaling of a Saddle-Node Bifurcation on a Limit Cycle Disclosed in a Semiconductor Experiment

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During low-temperature impact ionization breakdown in extrinsic germanium, spontaneous current and voltage oscillations can be observed. The onset of periodic oscillatory behavior is governed by a saddle-node bifurcation on a limit cycle. Besides the well-known square-root law of control parameter dependence, we find a different scaling in the vicinity of the bifurcation point. Such anomalous behavior is reproduced by a simple model approach.

Electric avalanche breakdown caused by impact ionization of shallow impurities in p-type germanium at liquid-helium temperatures provides a challenging experimental system capable to evaluate theoretical predictions for nonlinear dynamical phenomena [1]. In what follows, we concentrate on a nonhysteretic transition from a stable fixed point to a limit cycle (characterized by a saddle-node bifurcation on a limit cycle [2]). The measured scaling of the frequency and amplitude of spontaneous oscillations as well as the underlying topology of the attractor have been analyzed previously [3]. The pronounced deviation from the predicted square-root frequency scaling in the vicinity of the bifurcation point is subject of the present investigation.

Our experimental system consists of single-crystalline p-type germanium, electrically driven to low-temperature impact ionization breakdown. The sample of the dimension $0.25 \times 2.0 \times 4.4$ mm^3 and an acceptor impurity concentration of about 10^{14} cm^{-3} is furnished with ohmic contacts and connected in series with a load resistor and a constant voltage source. The temperature of the liquid-helium bath was kept at $T = 1.89$ K, where nearly all charge carriers are frozen out. Electric breakdown due to impact ionization of shallow impurities by hot charge carriers takes place at field values of typically a few V/cm, causing current-voltage characteristics with S-shaped negative differential resistance. In the not fully developed breakdown region, plasma-like current filaments arise together with spontaneous oscillations as dissipative structures. These findings can be explained, in principle, by semiconductor physics treating generation and recombination processes. The detailed structure of the current and voltage oscillations shows a high sensitivity against smallest changes of the experimental control parameters (namely, the temperature, the time-averaged current, and the external magnetic field oriented perpendicular to the direction of the electric field) [1, 4].

For the transition investigated, all parameters except the magnetic field were kept constant. We start from a nonoscillatory state. With decreasing strength of the magnetic field applied, we surpass a threshold where an oscillation with finite amplitude sets in abruptly. It remains constant under further variation of the control parameter (Figure 1). In contrast to the well-known Hopf bifurcation, the present transition starts with zero frequency. The scaling plotted in Fig. 2 develops toward a square-root law (indicated by the straight line). So far, these results confirm the model description put forward in [2] for the case of a saddle-node bifurcation on a limit cycle. However, close to the bifurcation point the smallest frequency values deviate from the predicted scaling – even though best accordance should be found there.

At this point, analyzing the series of attractor reconstructions shown in Fig. 3 comes to the rescue. Under increase of the magnetic field, the originally smooth limit cycle (1) develops a sharp bend (2–4), which finally hits the diagonal (5) for applying the parameter

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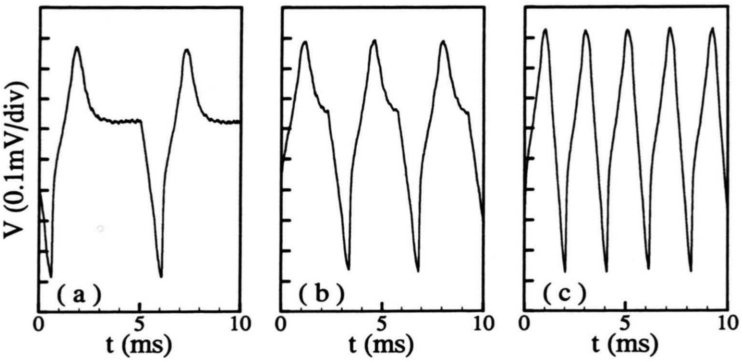

Fig. 1. Temporal structure of spontaneous voltage oscillations obtained at different magnetic field (a) $B=-0.176$ mT, (b) $B=-0.170$ mT, (c) $B=-0.076$ mT, and the constant parameters time-averaged current $I=2.8405$ mA and temperature $T=1.89$ K. The negative sign of the magnetic field strength indicates its reverse orientation compared to [3].

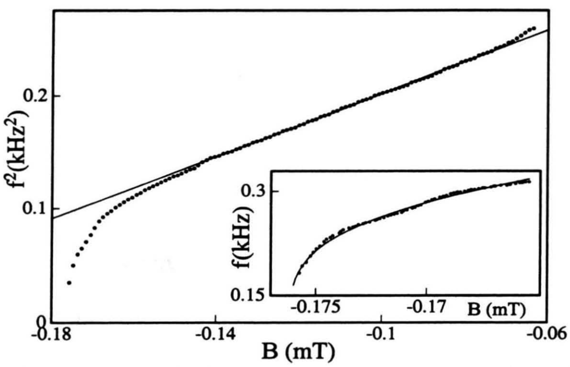

Fig. 2. Square of the frequency versus magnetic field obtained at the constant parameters of Figure 1. The straight line corresponds to a square-root scaling. The inset displays the logarithmic scaling of a close-up near the bifurcation point. The solid curve derives from a fit with $f=c\,\dfrac{1}{-\log(B-B_c)}$ and $c=1.4443\ \mathrm{V/m^2}$, $B_c=-0.17616$ mT. The full dots indicate the measured data.

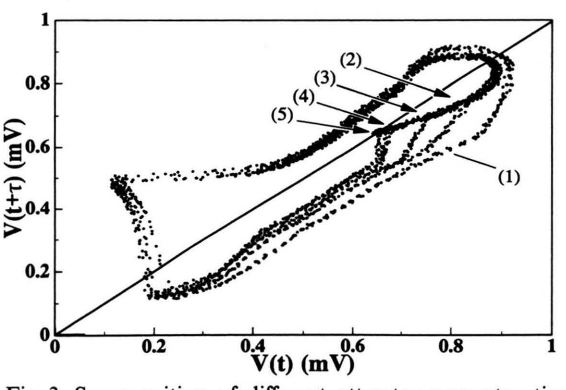

Fig. 3. Superposition of different attractor reconstructions obtained by the time-delay method ($\tau=0.2$ ms) from spontaneous voltage oscillations at different magnetic field $B=-0.076$ mT (1), $B=-0.126$ mT (2), $B=-0.145$ mT (3), $B=-0.162$ mT (4), $B=-0.176$ mT (5) and the constant parameters of Figure 1.

value of the bifurcation point. The distinct sharpness of the bend structure observed is in remarkable contrast to a parabolic line that approaches the diagonal in case of the standard saddle-node bifurcation. In order to demonstrate that the anomalous scaling behavior has its origin in a modified saddle-node bifurcation, we analyze the map $x_{n+1}=f(x_n)$ modulo 1. The design of the piecewise linear function $f(x)$ was guided by the series of experimentally constructed attractors presented in Figure 3. We use

$$f(x)=\begin{cases} x-b & 0\le x<\mu \\ a(x-0.5)+0.5 & \mu\le x\le 1 \end{cases}, \qquad (1)^*$$

where $0<a,\,b\le 1$ are constants and $0\le\mu<1$ is the control parameter. At $\mu=\mu_c=0.5$, the fixed point at $x=0.5$ becomes unstable. For $\mu>\mu_c$, the values of x_n oscillate. Their frequency follows the relation

$$f\propto\frac{1}{-\log(\mu-\mu_c)}\,\frac{1}{\log a}. \qquad (2)$$

Such a logarithmic scaling fits well to the frequency dependence measured in the vicinity of the bifurcation point (see inset of Figure 2).

Finally, we point out that the present modified saddle-node bifurcation and its anomalous scaling behavior should be found also for the case of type-I intermittency.

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* A map similar to (1) and its logarithmic scaling for the case of intermittency have been discussed recently by M. Bauer et al., Phys. Rev. Lett. **68**, 1625 (1992) and D. R. He et al., Phys. Lett. **A 171,** 61 (1992).

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